Sec. 1.2 General and Particular Sols

If ODE can be written in the form

$$\frac{dy}{dx} = f(x)$$

$f(x)$ is some expression with no $y, y', \dots$

then the solutions $y(x)$ are just

$$y(x) = \int f(x)\,dx + C \qquad (C \text{ unknown const.})$$

If solution $y(x)$ involves unknown const. C, it is a general solution.

Ex: $y' = 12x^2(y^2+1)$, $\rightarrow$ $y(x) = \tan(4x^3 + C)$ is a gen. sol. for the ODE.

If we also have initial value (ex: $y(0) = -1$, or $y(0) = y_0$.) then we find explicit C.

$$\text{IVP} \begin{cases} y' = 12x^2(y^2+1) \\ y(0) = 1 \end{cases} \implies y(x) = \tan\left(4x^3 + \frac{\pi}{4}\right)$$

This solution is a particular sol. (no C)

Ex Find a part. sol. to IVP $\begin{cases} y' = 2x + 3 \\ y(1) = 2 \end{cases}$

Gen sol: $y' = 2x + 3$

$$\implies \frac{dy}{dx} = 2x + 3$$

$$\implies dy = (2x+3)\,dx$$

$$\Rightarrow \int dy = \int (2x+3)\,dx$$

$$y + c_1 = x^2 + 3x + c_2$$

$$y = x^2 + 3x + \underbrace{\boxed{c_2 - c_1}}_{\text{"c"}}$$

So

$$y(x) = x^2 + 3x + c$$

is a general sol. for the ODE.

(Read "unknown constants" note)

<u>Part. solution</u> : need $y(1) = 2$

$$\Rightarrow 2 = y(1) = 1 + 3 + C = 4 + C$$

$$\Rightarrow C = -2,$$

so $y(x) = x^2 + 3x - 2$ is the part. sol.

<u>Ex</u> Find a general solution to the ODE

$$\frac{dy}{dx} = \frac{12}{x^2+16}$$

<u>Recall</u> $\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{(x/a)^2+1} \cdot (1/a) = \frac{a}{x^2+a^2}$

$$dy = \frac{3 \cdot 4}{x^2+4^2}\,dx$$

$$\Rightarrow y = 3\int \frac{4}{x^2+(4)^2}\,dx + C$$

$$\Rightarrow y = 3\tan^{-1}(x/4) + C$$

is a general solution.

<u>Ex (cont.)</u> : Find a particular sol. having

$$y(0) = 1.$$

$$\Rightarrow 1 = y(0) = 3\tan^{-1}(0) + C = C,$$

so $y(x) = 3\tan^{-1}\left(\frac{x}{4}\right) + 1$ is a part. sol. to the IVP $\begin{cases} y' = \frac{12}{x^2+16} \\ y(0) = 1 \end{cases}$.